FORMULARIUM VOOR GONIOMETRIE

1 De goniometrische getallen

$\sin\alpha = y_P$

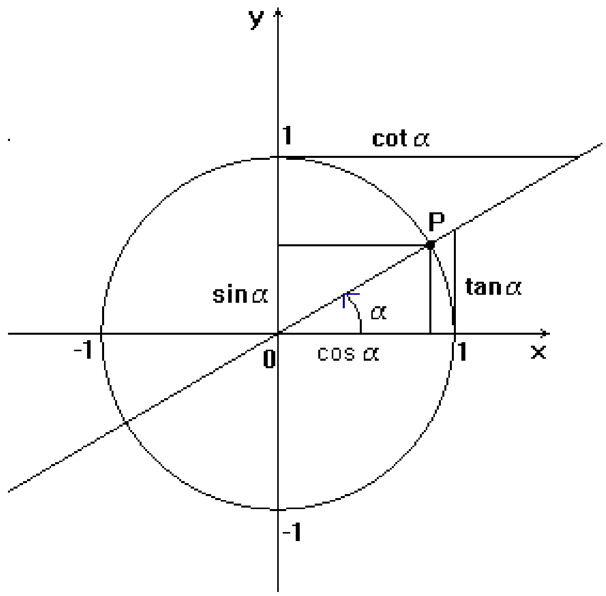
$\cos\alpha = x_P$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{1}{\tan\alpha}$

$\sec\alpha = \dfrac{1}{\cos\alpha}$

$\operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha}$



2 Grondformule : $\sin^2\alpha + \cos^2\alpha = 1$

Gevolgen : $\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$ en $\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$

$1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha}$ en $1+\cot^2\alpha = \dfrac{1}{\sin^2\alpha}$

$\sin^2\alpha = \dfrac{\tan^2\alpha}{1+\tan^2\alpha}$ en $\cos^2\alpha = \dfrac{1}{1+\tan^2\alpha}$

3 Hoekeenheden : zestigdelige graden en radialen

1 radiaal = de grootte van een middelpuntshoek die op een boog staat die even lang is als de straal van de cirkel

$360° = 2\pi$ rad en 1 rad = $\dfrac{180°}{\pi}$ = 57° 17′ 45″

$\alpha° = \alpha \cdot \dfrac{\pi}{180}$ rad en α rad = $\alpha \cdot \dfrac{180°}{\pi}$

0°	30°	45°	60°	90°	120°	135°	150°	180°
0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	π

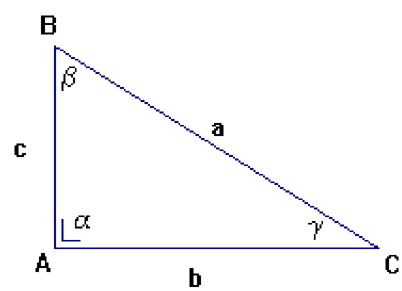
4 Courante goniometrische waarden

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-
cot	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

5 Oplossen van driehoeken

- RECHTHOEKIGE DRIEHOEKEN

$\sin \alpha = \sin 90° = 1$
$\sin \beta = b/a$ en $\cos \beta = c/a$
$\tan \beta = b/c$
$\sin \gamma = c/a$ en $\cos \gamma = b/a$
$\tan \gamma = c/b$

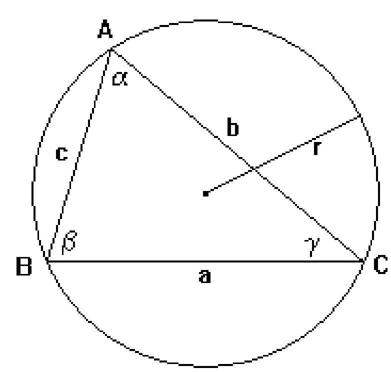


Stelling van Pythagoras : $a^2 = b^2 + c^2$

- WILLEKEURIGE DRIEHOEKEN

$\alpha + \beta + \gamma = 180°$

sinusregel : $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2r$



cosinusregel : $a^2 = b^2 + c^2 - 2bc.\cos\alpha$
$b^2 = c^2 + a^2 - 2ac.\cos\beta$
$c^2 = a^2 + b^2 - 2ab.\cos\gamma$

Opp. Δ ABC = $\frac{1}{2}bc.\sin\alpha$ = $\frac{1}{2}ca.\sin\beta$ = $\frac{1}{2}ab.\sin\gamma$

Opp. Δ ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

waarbij $s = \frac{a+b+c}{2}$ (formule van Heron)

6 Verwante hoeken

- Hoeken die een geheel veelvoud van 360° verschillen :

$\sin(\alpha + k.360°) = \sin \alpha$
$\cos(\alpha + k.360°) = \cos \alpha$
$\tan(\alpha + k.360°) = \tan \alpha$
$\cot(\alpha + k.360°) = \cot \alpha$

- Complementaire hoeken : som = 90°

$\sin (90°-\alpha) = \cos \alpha$
$\cos (90°-\alpha) = \sin \alpha$
$\tan (90°-\alpha) = \cot \alpha$
$\cot (90°-\alpha) = \tan \alpha$

- Supplementaire hoeken : som = 180°

$\sin (180°-\alpha) = \sin \alpha$
$\cos (180°-\alpha) = - \cos \alpha$
$\tan (180°-\alpha) = - \tan \alpha$
$\cot (180°-\alpha) = - \cot \alpha$

- Tegengestelde hoeken : som = 0° en hoeken waarvan de som = 360°

$\sin(-\alpha) = - \sin \alpha$
$\cos(-\alpha) = \cos \alpha$
$\tan(-\alpha) = - \tan \alpha$
$\cot(-\alpha) = - \cot \alpha$

$\sin(360°-\alpha) = - \sin \alpha$
$\cos(360°-\alpha) = \cos \alpha$
$\tan(360°-\alpha) = - \tan \alpha$
$\cot(360°-\alpha) = - \cot \alpha$

- Antisupplementaire hoeken : verschil = 180°

$\sin(180°+\alpha) = - \sin \alpha$
$\cos(180°+\alpha) = - \cos \alpha$
$\tan(180°+\alpha) = \tan \alpha$
$\cot(180°+\alpha) = \cot \alpha$

- Anticomplementaire hoeken : verschil = 90°

$\sin(90°+\alpha) = \cos(-\alpha) = \cos \alpha$
$\cos(90°+\alpha) = \sin(-\alpha) = - \sin \alpha$
$\tan(90°+\alpha) = \cot(-\alpha) = - \cot \alpha$
$\cot(90°+\alpha) = \tan(-\alpha) = - \tan \alpha$

7 Som- en verschilformules

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$
$$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha.\tan\beta}$$

8 Formules voor de dubbele hoek

$$\sin 2\alpha = 2.\sin\alpha.\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha-\sin^2\alpha$$
$$\cos 2\alpha = 2.\cos^2\alpha-1 \quad \textit{of} \quad \textit{ook} : 2.\cos^2\alpha = 1+\cos 2\alpha$$
$$\cos 2\alpha = 1-2.\sin^2\alpha \quad \textit{of} \quad \textit{ook} : 2.\sin^2\alpha = 1-\cos 2\alpha$$
$$\tan 2\alpha = \frac{2.\tan\alpha}{1-\tan^2\alpha}$$

9 Formules voor de drievoudige hoek

$$\sin 3\alpha = 3.\sin\alpha-4.\sin^3\alpha$$
$$\cos 3\alpha = 4.\cos^3\alpha-3.\cos\alpha$$

10 Formules van Simpson

$$\sin p+\sin q = 2.\sin\frac{p+q}{2}.\cos\frac{p-q}{2}$$
$$\sin p-\sin q = 2.\cos\frac{p+q}{2}.\sin\frac{p-q}{2}$$
$$\cos p+\cos q = 2.\cos\frac{p+q}{2}.\cos\frac{p-q}{2}$$
$$\cos p-\cos q = -2.\sin\frac{p+q}{2}.\sin\frac{p-q}{2}$$

Omgekeerde formules van Simpson

$$\sin p.\cos q = \frac{1}{2}[\sin(p+q) + \sin(p-q)]$$

$$\cos p.\cos q = \frac{1}{2}[\cos(p+q) + \cos(p-q)]$$

$$\sin p.\sin q = -\frac{1}{2}[\cos(p+q) - \cos(p-q)]$$

11 Formules van Carnot

$$1 + \cos\alpha = 2.\cos^2\frac{\alpha}{2}$$

$$1 - \cos\alpha = 2.\sin^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \quad \textit{of ook}: \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \quad \textit{of ook}: \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

12 De t-formules

Stel $\tan\frac{\alpha}{2} = \text{t}$, dan is

$$\sin\alpha = \frac{2.t}{1+t^2}$$

$$\cos\alpha = \frac{1-t^2}{1+t^2}$$

$$\tan\alpha = \frac{2.t}{1-t^2}$$